Effect of Interfacial Drag on Gas Absorption with Chemical Reaction in a Vertical Tube

MOHAMMAD-REZA RIAZI

Department of Chemical Engineering
The Pennsylvania State University
University Park, PA 16802

AMIR FAGHRI

Department of Engineering
Wright State University
Dayton, OH 45435

The problem of gas absorption with chemical reaction has been studied extensively, and investigators in recent years (Stepanek and Achwal, 1976; Pedersen and Prenosil, 1981) have shown the effect of a finite gas-side resistance on absorption rate. However, in all the analyses made by these investigators, a parabolic velocity distribution in the liquid film with no shear stress at the gas-liquid (G–L) interface was assumed, and the effect of shear stress at the G–L interface on velocity distribution in the liquid film was neglected. The purpose of this study is to show the effect of interfacial shear stress on the rate of absorption from a gas phase to a laminar falling liquid film in a vertical tube. In our analysis we assume that the physical properties of liquid are constant; in addition, the curvature of the layer and the interfacial surface waves are neglected. The assumption of neglecting curvature is valid as long as the layer thickness, δ, is small in comparison to the tube diameter, d. Furthermore we assume that the gas phase concentration and the film thickness are constant at a given liquid loading. Under these circumstances the transport equation describing concentration in laminar liquid film with first-order chemical reaction (neglecting the axial dispersion term) in appropriate dimensionless variables becomes:

$$\bar{u}_L \frac{\partial \bar{C}}{\partial \bar{z}} = \frac{\partial^2 \bar{C}}{\partial \bar{x}^2} - \alpha \bar{C} \tag{1}$$

The corresponding boundary conditions for Eq. 1 for a finite gas-side resistance are:

$$\bar{z} = 0, \quad 0 \le \bar{x} \le 1; \quad \bar{C} = 0 \tag{2}$$

$$\bar{x} = 0, \quad \bar{z} > 0; \quad \frac{\partial \bar{C}}{\partial \bar{x}} = N(\bar{C} - 1) \tag{3}$$

$$\bar{x} = 1, \quad \bar{z} > 0; \quad \frac{\partial \bar{C}}{\partial \bar{x}} = 0 \tag{4}$$

Figure 1 shows the physical model used. Liquid enters the tube on the top with the thickness of δ. Gas may enter from the top (concurrent) or from the bottom in a countercurrent flow. Pedersen and Prenosil (1981) solved this problem by assuming parabolic distribution for $\bar{u}_L$.

In this analysis we do not neglect the interfacial shear stress at the G–L interface, τ_i, and upon integration of the momentum equation we find the following velocity distribution for the liquid film in a dimensionless form:

$$\bar{u}_L = (1 - \bar{x}^2) + J\gamma(1 - \bar{x}) \tag{5}$$

where

$$J = \begin{cases} +1 & \text{concurrent flow} \\ -1 & \text{countercurrent flow} \end{cases}$$

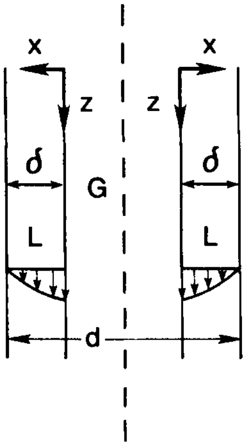

Figure 1. Diagram of the absorption process showing the coordinate system considered.

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The Reynolds number for the liquid phase is obtained as:

$$R_{eL} = \frac{4\Gamma}{\mu_L} = \lambda\left(\frac{4}{3} + 2J\gamma\right) \tag{6}$$

As discussed by Bird et al. (1960), the shear stress at the G–L interface, τ_i, can be written as:

$$\tau_i = \tau_f + \tau_d$$

or in a dimensionless form as:

$$\gamma = \gamma_f + \gamma_d \tag{7}$$

where γ_f is the dimensionless shear stress due to friction at the interface and γ_d is the dimensionless shear stress due to mass transfer from the gas phase to the liquid phase. If C_f is the friction coefficient and $\dot{\overline{m}}_d$ is the mass transfer rate, Eq. 7 can be written as:

$$\gamma = \frac{C_f}{2}\left(\frac{\rho_G}{\rho_L}\right)\left(\frac{\lambda}{4}\right)(\overline{u}_G - J\overline{u}_{Li})^2 + \frac{R}{2}\dot{\overline{m}}_d(\overline{u}_G - J\overline{u}_{Li}) \tag{8}$$

where

$$\dot{\overline{m}}_d = -\left.\frac{\partial\overline{C}}{\partial\overline{x}}\right|_{\overline{x}=0}$$

In Eq. 8, $\overline{u}_G$ is the gas velocity and can be obtained from the gas Reynolds number as

$$\overline{u}_G = \left(\frac{2}{\lambda}\right)\left(\frac{\nu_G}{\nu_L}\right)\frac{R_{eG}}{\overline{d} - 2.0} \tag{9}$$

$\overline{u}_{Li}$ is the liquid velocity at the interface and is obtained from Eq. 5 at $\overline{x} = 0$

$$\overline{u}_{Li} = \overline{u}_L|_{\overline{x}=0} = 1 + J\gamma \tag{10}$$

The coefficient C_f was used from the correlation of data for fully developed annular gas-liquid flow. The coefficient accounts for the irregularities at the surface of the liquid layer as well as pressure gradient. The single phase friction coefficient in terms of Re_G is:

$$\frac{C_f}{2} = \begin{cases} \dfrac{8}{Re_G} & Re_G < 2{,}000 \\ \dfrac{Re_G^{0.33}}{3{,}050} & 2{,}000 < Re_G < 4{,}000 \\ \dfrac{0.04}{Re_G\ 0.25} & Re_G < 4{,}000 \end{cases} \tag{11}$$

Henstock and Hanratty (1976) suggested that the friction factor for $Re_G < 4{,}000$ given by Eq. 11 should be multiplied by a term $(1 + 1{,}400\ F)$ in order to account for the two-phase flow nature where for the laminar liquid film

$$F = \frac{\beta(Re_L)}{Re_G{}^{0.9}}\frac{\nu_L}{\nu_G}\frac{\rho_L}{\rho_G}[1 - \exp(-\gamma)] \tag{12}$$

in which

$$\beta = [(0.707\ Re^{1/2})^{2.5} + (0.0379\ Re_L^{0.9})^{2.5}]^{0.4}$$

After evaluation of γ from Eq. 8, Eq. 5 can be used to obtain the velocity distribution in the liquid phase. Once the velocity in the liquid film is found, one can obtain the concentration distribution from Eq. 1. In order to show the influence of interfacial shear stress on absorption rate, a yield factor E is defined as follows:

$$E = \frac{\displaystyle\int_0^{\overline{z}}\left(-\left.\frac{\partial\overline{C}}{\partial\overline{x}}\right|_{\overline{x}=0}\right)d\overline{z}}{\displaystyle\int_0^{\overline{z}}\left(-\left.\frac{\partial\overline{C}}{\partial\overline{x}}\right|_{\substack{\overline{x}=0\\ \gamma=0}}\right)d\overline{z}} \tag{13}$$

By evaluating the factor E from Eq. 13, the effect of the interfacial drag on the absorption rate for different cases can be studied. Equation 1 with the corresponding boundary conditions was solved numerically using an implicit finite-difference method yielding a tridiagonal matrix. A grid size of 400 in the $\overline{x}$ direction was chosen, and using step-by-step advancing technique in the $\overline{z}$ direction, values of $\overline{C}$ were obtained. Analytical results from Pedersen and Prenosil (1981) were used to check our numerical results for the case of no interfacial drag. The agreement for both the concentration distribution and the absorption rate was satisfactory (less than 1% difference) for different values of α and N.

In order to obtain concentration distributions from Eq. 1, the first step is to determine the interfacial shear stress from Eq. 8. However, in order to determine γ, the concentration gradient at the interface, $\dot{\overline{m}}_d$ is needed, therefore an iteration process is required.

RESULTS

The numerical calculations were carried up to 3.00 m tube length for $d = 2.4 \times 10^{-2}$ m; $U_L = 10^{-6}$ m²/s; $\rho_L = 10^3$ kg/m³; $D = 10^{-9}$ m²/s; $\rho_G/\rho_L = 0.0013$; $\nu_G/\nu_L = 15$; $\alpha = 0.1$; and $R = 10^{-6}$. From the numerical results it was found that γ_d is negligible in comparison with γ_f. Negligible values of γ_d were anticipated, since in the absorption processes the rate of mass transfer is not significant. Neglecting γ_d simplifies our calculations since the solution of Eq. 1 does not enter the iteration loop for obtaining the velocity distribution. In the calculations, $\gamma = 33.1$ is equivalent to the liquid film thickness of 1.5×10^{-4} m.

Figure 2 shows how the interfacial drag affects the concentration distribution for $Re_G = 15{,}000$. Concentration at two different lengths, $\overline{z} = 0.25$ and $\overline{z} = 1.2$, is shown. $\overline{z} = 1.2$ is equivalent to a tube length of 3.0 m when $\gamma = 33.1$. Figure 3 shows the average concentration $\overline{C}_{avg}$ for different gas Reynolds numbers, defined as follows:

$$\overline{C}_{avg} = \frac{\displaystyle\int_0^1 \overline{u}_L\,\overline{C}\,d\overline{x}}{\displaystyle\int_0^1 \overline{u}_L\,d\overline{x}} \tag{14}$$

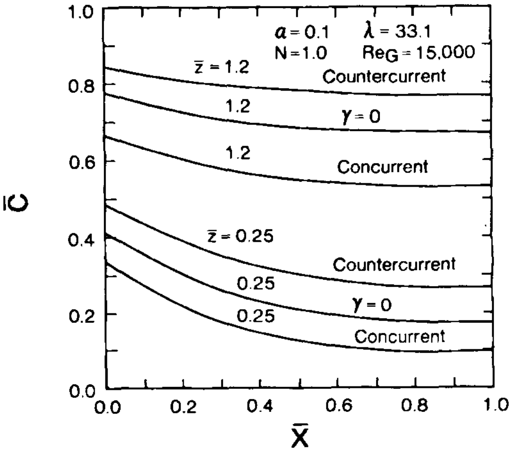

Figure 2. Concentration profile for concurrent and counter current flows.

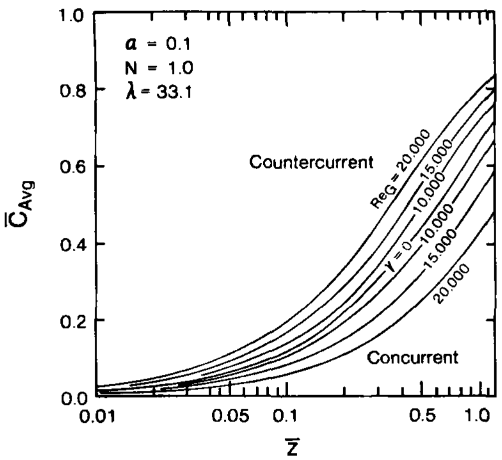

Figure 3. Average concentration along the tube length for different gas Reynolds numbers in concurrent and countercurrent flow.

Concentrations at a given length in concurrent flow are less than the concentrations in countercurrent flow. Figure 4 shows the yield factor E defined by Eq. 13 for different gas Reynolds numbers. *The line* $E = 1$ *corresponds to the case of* $\gamma = 0$. When the flow is concurrent, values of E are higher than 1, indicating that the gas absorption rate has been increased by including the interfacial drag in the analysis. For $Re_G < 5{,}000$, E is close to unity and in fact the effect of interfacial shear stress can be neglected. For countercurrent flow values of E are less than 1. In Figure 5 the effect of gas Reynolds number on liquid Reynolds number is shown. Figure 6 shows the effect of gas-side mass transfer resistance (parameter N) on the yield factor E. When N increases, the absorption rate increases too and therefore the effect of γ on the absorption rate is enhanced. Comparison of Figures 4 and 6 indicates how E increases as gas-side resistance decreases (N increases).

In summary, the above analysis indicates that neglecting interfacial shear stress in gas absorption problems, especially at gas Reynolds numbers of greater than 5,000, is not reasonable. In concurrent flow, shear stress causes an increase on the rate of absorption, while in countercurrent flow the interfacial shear stress causes a decrease in the absorption rate.

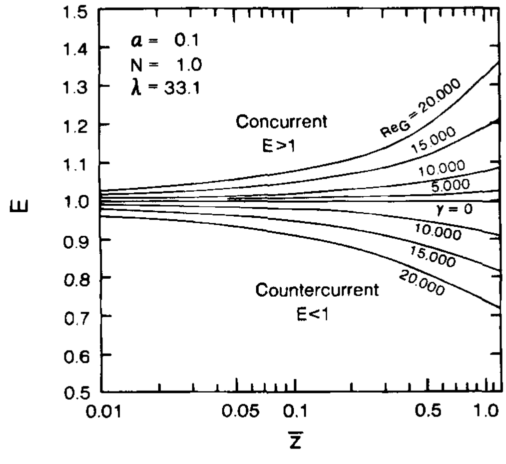

Figure 4. Variation of factor *E* along the tube length for *N* = 1.0 and λ = 33.1.

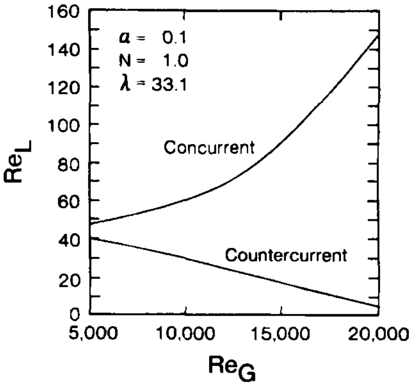

Figure 5. Variation of liquid Reynolds numbers with gas Reynolds numbers for λ = 33.1.

NOTATION

C = concentration in the liquid phase
C^* = concentration in equilibrium with the bulk of the gas
$\bar{C}$ = dimensionless concentration, $\bar{C} = C/C^*$
$\bar{C}_{avg}$ = dimensionless average concentration, defined by Eq. 14
C_f = friction factor
D = diffusivity
d = diameter of the tube
$\bar{d}$ = dimensionless diameter of the tube, $d = \bar{d}/\delta$
E = dimensionless factor showing the effect of interfacial drag on the absorption rate defined in Eq. 13
F = dimensionless group containing flow rates and fluid properties, defined in Eq. 12
g = acceleration of gravity
J = parameter defined in Eq. 5
k = rate constant for a first-order reaction
k_G = mass transfer coefficient in the gas in terms of liquid concentration
$\dot{\bar{m}}_d$ = dimensionless local mass transfer rate
N = dimensionless parameter, $N = k_G\delta/D$

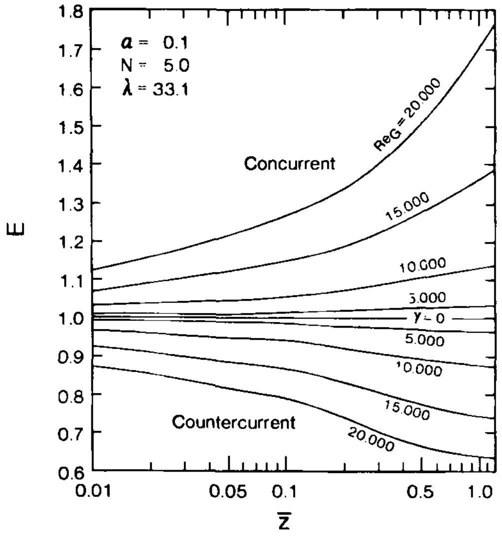

Figure 6. Variation of factor *E* along the tube length for *N* = 5.0 and λ = 33.1.

R = dimensionless parameter, $R = DC^*/\mu_L$
Re_G = gas Reynolds number
Re_L = liquid Reynolds number, defined in Eq. 6
u_G = gas velocity
u_L = liquid velocity
u_{Li} = liquid velocity at the surface
u^0_{Li} = liquid velocity at the interface with zero interfacial drag, $u^0_{Li} = g\delta^2/2\nu_L$
$\bar{u}_G$ = dimensionless gas velocity, $\bar{u}_G = u_G/u^0_{Li}$
$\bar{u}_L$ = dimensionless liquid velocity, $\bar{u}_L = u_L/u^0_{Li}$
$\bar{u}_{Li}$ = dimensionless liquid velocity at the interface, $\bar{u}_{Li} = u_{Li}/u^0_{Li}$
x = distance from the G–L interface toward the wall
$\bar{x}$ = dimensionless distance, $\bar{x} = x/\delta$
z = coordinate in the direction of flow
$\bar{z}$ = dimensionless distance, $\bar{z} = Dz/u^0_{Li}\delta^2 = (D/\nu_L)(2/\gamma) z/\delta$

Greek Letters

α = dimensionless reaction constant, $\alpha = k\delta^2/D$
β = function of liquid film Reynolds number, defined in Eq. 12
γ = dimensionless interfacial shear stress, $\gamma = \tau_i/\rho_L g\delta$
γ_d = dimensionless interfacial shear stress due to mass transfer, $\gamma_d = \tau_d\rho_L g\delta$
γ_f = dimensionless interfacial shear stress due to friction, $\gamma_f = \tau_f/\rho_L g\delta$
δ = liquid film thickness
λ = dimensionless film thickness parameter, $\lambda = g\delta^3/\nu_L^2$
Γ = liquid loading
ρ_G = gas density
ρ_L = liquid density
μ_G = gas viscosity
μ_L = liquid viscosity
ν_G = gas kinematic viscosity
ν_L = liquid kinematic viscosity
τ_d = interfacial shear stress due to mass transfer
τ_f = interfacial shear stress due to friction
τ_i = interfacial shear stress

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